PACO : Vers une meilleure paramétrisation de la côte et des conditions limites dans les modèles d'océan

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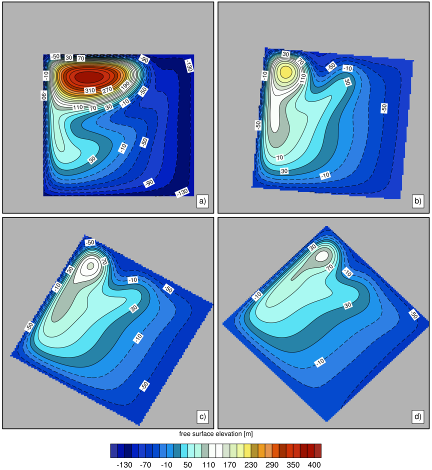
LJK/INRIA, AirSea, Grenoble

Contribuer à la compréhension fine des interactions entre représentation discrétisée de la côte, profils analytiques de couches limites latérales, et choix de discrétisation des conditions aux limites en s'appuyant sur :

- calculs analytiques (prenant en compte la forme continue de la ligne de côte dans la formulation des conditions aux limites)
- contrôle optimal des schémas numériques au voisinage de la côte.

LEFE-GMMC, LEFE-MANU



Solution instantanée après 8 ans d'intégration dans le cas free-slip pour un angle de

0° (a), 5° (b), 30° (c) et 45° (d)

Code *shallow water* 2D

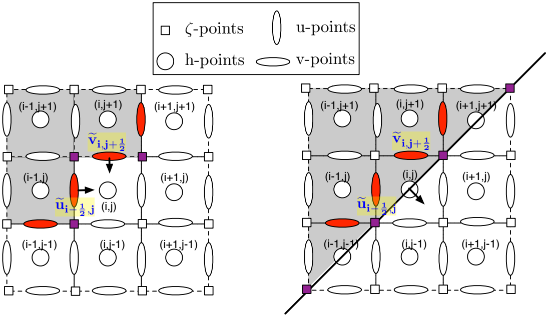
- Schéma en temps : RK4
- Advection : forme vecteur-invariant (enstrophy conserving)
- Viscosité : forme vorticité-divergence

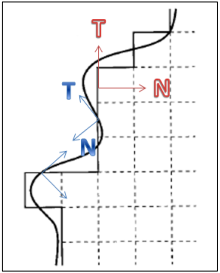
$$\begin{aligned} \nu\nabla^2 u &\rightarrow \nu\left(\partial_x D - \partial_y \zeta\right) \\ \nu\nabla^2 v &\rightarrow \nu\left(\partial_x \zeta + \partial_y D\right) \end{aligned}$$

Paramètres du problème

- 1750 km $\times$ 1750 km ($\Delta x = 10$ km)
- $\nu = 500$ m^2 s^{-1}
- $g = 0.01$ m s^{-2}
- $r_D = 10^{-7}$s^{-1}

Cas particulier : rotation à 45° (see [3])





Conditions limites : $\mathbf{u}\cdot\mathbf{n}=0$, $\partial\mathbf{u}/\partial\mathbf{n}=0$

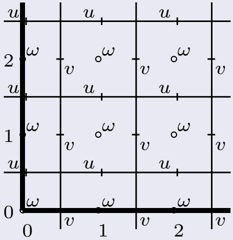
Imperméabilité $\Rightarrow \zeta = \left(\dfrac{\mathbf{u}}{R} - \dfrac{\partial\mathbf{u}}{\partial\mathbf{n}}\right)\cdot\boldsymbol{\tau} \Rightarrow$ glissement $+\ R\to\infty\ :\ \zeta=0$

$$\left\{\begin{array}{lcl} \tilde{u}_{i+\frac{1}{2},j+1} + u_{i+\frac{1}{2},j} & = & \tilde{v}_{i,j+\frac{1}{2}} + v_{i+1,j+\frac{1}{2}} \\ \tilde{u}_{i+\frac{1}{2},j+1} + \tilde{v}_{i,j+\frac{1}{2}} & = & u_{i+\frac{1}{2},j} + v_{i+1,j+\frac{1}{2}} \end{array}\right. \Rightarrow \left\{\begin{array}{lcl} \tilde{u}_{i+\frac{1}{2},j+1} & = & v_{i+1,j+\frac{1}{2}} \\ \tilde{v}_{i,j+\frac{1}{2}} & = & u_{i+\frac{1}{2},j} \end{array}\right.$$

$$D_{i,j} = \frac{u_{i+\frac{1}{2},j}}{\Delta x} - \frac{v_{i,j-\frac{1}{2}}}{\Delta y} \to D_{i,j} = \left(\frac{u_{i+\frac{1}{2},j}}{\Delta x} + \frac{u_{i+\frac{1}{2},j}}{\Delta y}\right) - \left(\frac{v_{i,j-\frac{1}{2}}}{\Delta y} + \frac{v_{i,j-\frac{1}{2}}}{\Delta x}\right)$$

Layout:

- NEMO, Rectangular box, $1°/4$ resolution, single-gyre or double-gyre wind stress;
- Artificially generated data by the same model on the aligned grid;
- Assimilation of these data during 50 days interval;
- Analysis of the model on the 800 days interval



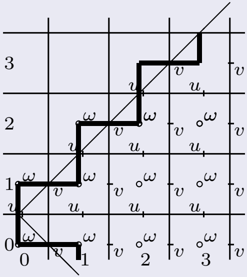


Figure: 45° rotated grid

Impermeability + free-slip boundary conditions: $(\vec{V},\vec{n}) = 0, \quad \dfrac{\partial(\vec{V},\vec{\tau})}{\partial \vec{n}} = 0$

Optimal Control of the Discrete Derivatives near the Boundary

Coefficients α are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$
\begin{aligned}
\left.\frac{\partial v}{\partial x}\right|_{\omega_b} &= \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h} \\
\left.\frac{\partial u}{\partial y}\right|_{\omega_b} &= \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h} \\
\left.\frac{\partial v}{\partial x}\right|_{\omega_b} &= \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h} \\
\left.\frac{\partial u}{\partial y}\right|_{\omega_b} &= \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h} \\
\left.\frac{\partial \omega}{\partial x}\right|_{v} &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h} \\
\left.\frac{\partial \omega}{\partial y}\right|_{u} &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}
\end{aligned}
$$

with the Initial Guess

$$
\begin{aligned}
\alpha_1 &= \alpha_2 = 0 \\
\alpha_1 &= -1, \quad \alpha_2 = 1 \\
\alpha_1 &= -1, \quad \alpha_2 = 1
\end{aligned}
$$

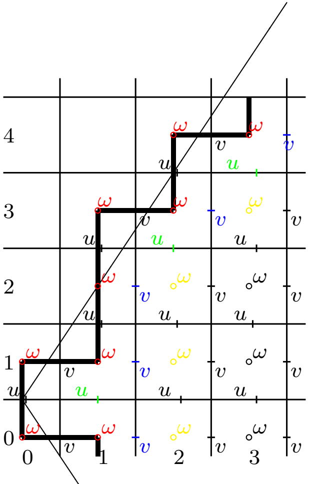


Figure: 30° rotated grid

The model: $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

$$
\begin{aligned}
J &= 10^{-4}(\|x(0) - x_{bgr}\|^2 + \|\alpha - \alpha_{bgr}\|^2) + \\
&+ \int_{t=0}^{T} t \int\int (u - u_{\mathrm{ref}})^2 + (v - v_{\mathrm{ref}})^2 + (ssh - ssh_{\mathrm{ref}})^2 dxdydt
\end{aligned}
$$

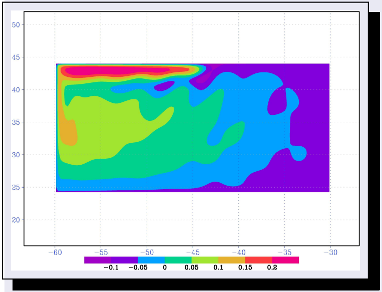
Layout:

- Joint control of the initial point $x(0)$ (interpolation errors) and the set of α;
- Artificially generated data by the same model on the aligned grid;
- Data Assimilation with the sequence of assimilation windows: 10, 30, 50 days with 30 iterations made in each window;
- Analysis of the solution on the 800 days interval.

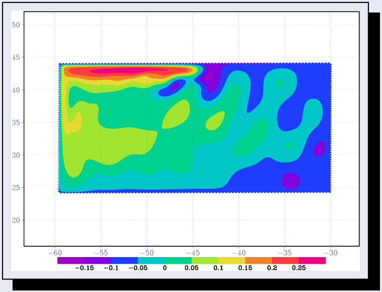
- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
- Adjoint is generated by Tapenade (Ecuador team, INRIA).

Single Gyre Control: 45° rotation, SSH

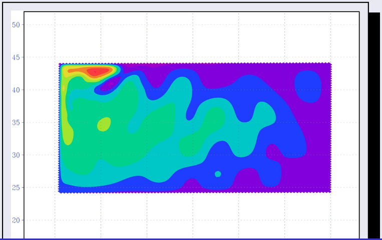
Reference, Optimal and Conventional BC 800 days later



Reference SSH

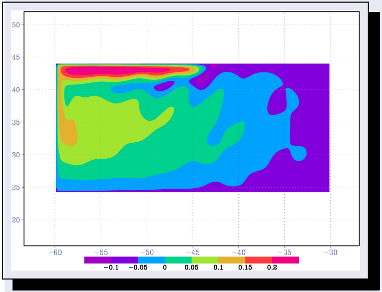


Rotated grid Optimal BC SSH

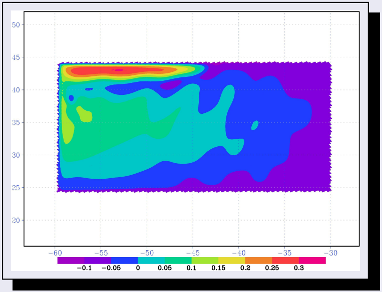


Single Gyre Control: 30° rotation, SSH

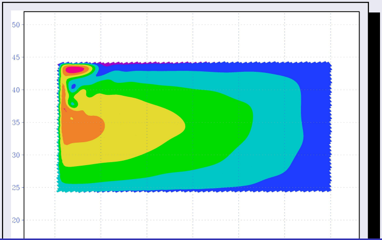
Reference, Optimal and Conventional BC 800 days later



Reference SSH



Rotated grid Optimal BC SSH



$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{v} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left.\frac{\partial \omega}{\partial y}\right|_{u} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

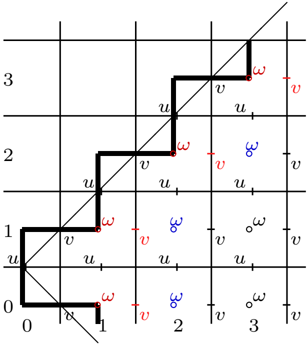


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$

$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

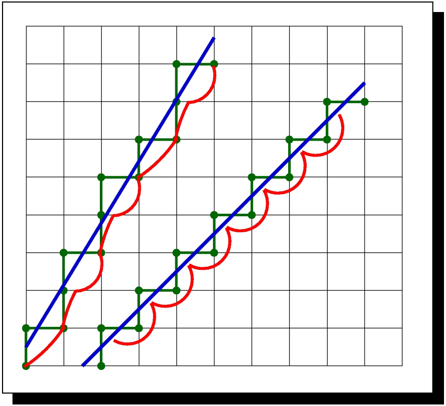
Optimized boundary

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

- Free-slip condition on a curvilinear boundary (see [4]): $\omega|_{bnd} = \frac{\vec{V}.\vec{\tau}}{R}$

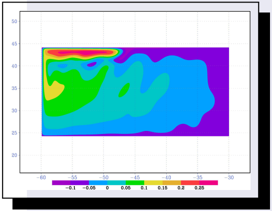
The optimized boundary is supposed to be a curvilinear one with:

- Constant $R_{45^\circ} = -h/\sqrt{2}$ for the 45° rotated grid,
- Variable R_{30° : $-h < R_{30^\circ} < 5h$ for the 30° rotated grid

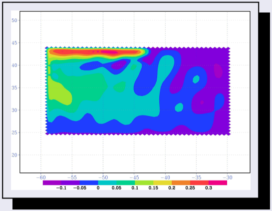


Physical boundary (blue), approximated by the grid (green) and optimized one (red).

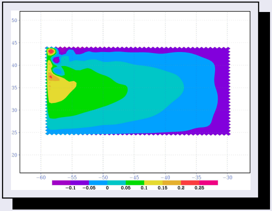
1°/2 resolution



Reference SSH

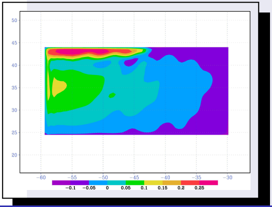


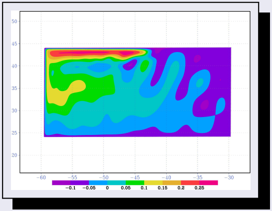
Rotated grid constant $R = -h/\sqrt{2}$ BC SSH

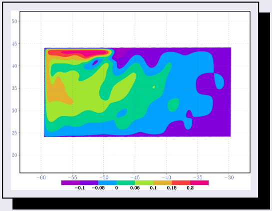


Rotated grid conventional BC SSH

1°/8 resolution







Conclusion

Boundary Conditions influence is important

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The physical meaning of the optimal boundary is difficult to understand;
- The set of α is not unique;
- The problem of identifiability is not addressed yet;
- The problem of stability is not even posed.

References:

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